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ASYMPTOTICALLY AUTONOMOUS DIFFERENTIAL EQUATIONS IN THE PLANE

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Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. We present a couple of examples where the solutions of asymptotically autonomous differential equations behave quite differently from the solutions of the corresponding limit equations. Nevertheless a Poincaré Bendixson type limit set trichotomy can be shown in the plane.

1. Introduction. An ordinary differential equation in $\mathbf{R}^n$,

$$\dot{x} = f(t, x), \tag{1.1}$$

is called *asymptotically autonomous*—with *limit equation*

$$\dot{y} = g(y), \tag{1.2}$$

if

$$f(t, x) \to g(x), \qquad t \to \infty, \quad \text{locally uniformly in } x \in \mathbf{R}^n,$$

i.e. for x in any compact subset of $\mathbf{R}^n$. For simplicity we assume that $f(t, x), g(x)$ are continuous functions and locally Lipschitz in x.

In an often quoted (and sometimes misquoted) paper, L. Markus [**23**] presents the following theorems concerning the ω-limit sets, $\omega(t_0, x_0)$, of forward bounded solutions x to (1.1), subject to $x(t_0) = x_0$,

$$\omega(t_0, x_0) = \bigcap_{s > t_0} \overline{\{x(t); t \geq s\}}.$$

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