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THE GREEN'S MATRIX FUNCTION AND RELATED EIGENVALUE RESULTS FOR A VECTOR DIFFERENCE EQUATION

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Dedicated to Paul Waltman on the occasion of his 60th birthday

We will derive a special form of a Green's matrix function for a second-order, self-adjoint vector difference equation, and then take advantage of this form in the application of cone theory in a Banach space to prove results concerning eigenvalues for a corresponding boundary value problem. The self-adjoint equation has been considered by Ahlbrandt and Hooker [**1**], and Peil and Peterson [**11**], among others. A discussion of the corresponding scalar self-adjoint equation appears in Kelley and Peterson [**9**]. Background on cone theory to difference equations can be found in Krasnosel'skiĭ [**10**] and Diaz [**2**]. Similar applications of cone theory to difference equations can be found in Hankerson and Henderson [**4**] and Hankerson and Peterson [**5, 6**]. Applications of cone theory to differential equations can be found in Eloe, Hankerson and Henderson [**3**].

We initially consider the second-order, self-adjoint vector difference equation

$$Ly(t) = -\Delta[P(t-1)\Delta y(t-1)] + Q(t)y(t) = 0 \tag{1}$$

on the discrete interval $[a+1, b+1] \equiv \{a+1, \dots, b+1\}$, where here $P(t)$ and $Q(t)$ are $n \times n$ matrix functions with $P(t)$ positive definite on $[a, b+1]$ and $Q(t)$ Hermitian on $[a+1, b+1]$. Solutions of (1) are defined on $[a, b+2]$.

If $y(t)$ is a complex solution of $Ly(t) = 0$, then on $[a+1, b+2]$,

$$y^*(t-1)P(t-1)y(t) - y^*(t)P(t-1)y(t-1) = c,$$

where c is complex constant and $*$ denotes the conjugate transpose. If $c = 0$, then $y(t)$ is called a prepared solution of (1), in which case $y^*(t-1)P(t-1)y(t)$ is real-valued on $[a+1, b+2]$.

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