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ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF A TWO TERM DIFFERENCE EQUATION

TIMOTHY PEIL AND ALLAN PETERSON

Dedicated to Paul Waltman on the occasion of his 60th birthday

We will be concerned with the $2n$-th order linear difference equation

$$Ly(t) \equiv \Delta^n[p(t-n)\Delta^n y(t-n)] + q(t)y(t) = 0 \tag{1}$$

where $p(t) > 0$ on the discrete interval $[a,\infty) \equiv \{a, a+1, \dots\}$ and where $q(t)$ is defined on the discrete interval $[a+n,\infty)$. Here Δ denotes the forward difference operator, i.e., $\Delta y(t) = y(t+1) - y(t)$. A function y defined on the discrete interval $[a,\infty)$ is a solution of (1), provided (1) holds for $t \geq a+n$.

There has been much recent interest in difference equations. See the recent books [**1, 4** and **7**–**9**] and the many references therein. Discrete time linear systems arise in discrete linear optimal control and filtering problems [**14**]. Cheng [**3**] studied equation (1) with $p(t) \equiv 1$ and $n = 2$. Smith and Taylor [**12**] studied a variation of equation (1) with $p(t) \equiv 1$, $n = 2$, and two additional lower order terms. We are also motivated by [**6**] and [**13**].

We now introduce *quasi-difference operators* so that the Lagrange identity of (1) has a nice form. For $0 \leq i \leq n-1$, define

$$\Delta_i y(t) = \Delta^i y(t),$$

and for $n \leq i \leq 2n-1$, define

$$\Delta_i y(t) = \Delta^{i-n}[p(t-i+n-1)\Delta^n y(t-i+n-1)].$$

One can then prove the Lagrange identity for (1).

Theorem 1. *For y and z defined on $[a,\infty)$,*

$$z(t)Ly(t) - y(t)Lz(t) = \Delta\{z(t); y(t)\}$$

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