ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 24, Number 1, Winter 1994

ON THE EXISTENCE OF MULTIPLE SOLUTIONS OF A BOUNDARY VALUE PROBLEM ARISING FROM FLOWS IN FLOATING CAVITIES

CHUNQING LU

Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. Existence of multiple solutions of the similarity equation $f''' + Q[Aff'' - f'^2] = \beta$ satisfying $f(0) = f(1) = f''(0) + 1 = f''(1) = 0$ is proved using the shooting method. Here Q, A and β are parameters, $Q > 0$ and $A = 1$.

1. Introduction. The third order nonlinear differential equation

$$f''' + Q[Aff'' - (f')^2] = \beta, \qquad f = f(\eta), \qquad 0 \leq \eta < 1$$

with boundary condition $f(0) = f(1) = f''(1) = f''(0) + 1 = 0$, where $Q > 0$, $A > 0$, and β are parameters, governs the velocity of boundary layer flow in a low Prandtl number fluid zone having the shape either of rectangular ($A = 1$) or a circular disk ($A = 2$) [**1, 2**]. Existence of solutions to the boundary value problem has been proved in [**4**] and [**5**] for the following cases:

(1) for given $A > 0$ and for $\beta \in [0, 1]$, there exists at least one $Q > 0$ for which the equation has at least one convex solution;

(2) Given $Q > 0$ and $A \in [1, 2]$, there exists at least one β for which the equation has a convex solution. Moreover, $\beta < 0$ if Q is sufficiently large;

(3) If $A = 2$, there exists a unique solution for every $Q > 0$;

(4) If $A = 1$, there may exist multiple solutions for some $Q > 0$.

In this paper we improve the result in (4). We present a proof of the existence of multiple solutions for $A = 1$ as long as Q is sufficiently large, i.e., if $A = 1$, then there exists a number $Q_0 > 0$ such that there are at least three solutions for any given $Q > Q_0$. Since Q

Received by the editors on March 9, 1993.