ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 24, Number 1, Winter 1994

GLOBAL STABILITY IN MODELS OF POPULATION DYNAMICS WITH DIFFUSION. II. CONTINUOUSLY VARYING ENVIRONMENTS

T. KRISZTIN AND H.I. FREEDMAN

Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. A class of models of single-species dynamics is considered where the diffusion coefficient and reaction term continuously depend on spatial position. It is shown that, under our prescribed conditions, there is a unique, positive, globally asymptotically stable steady state.

1. Introduction. Over the past several decades there have been thousands of papers written which deal with mathematical models and/or analyses of ecological dynamics. The majority of these papers consider dynamics in a closed environment with no diffusion and constant carrying capacities [**4**]. In nature, however, spatial effects quite often need to be considered. If one thinks of growth in a forest, the various species diffuse among the forest area. Further, changing soil conditions, elevations, foliage cover, etc., create a continuously changing carrying capacity.

A technique often employed by ecological field workers in aid of their analysis is to draw a transect through the environment and analyze the ecological dynamics along the transect, thus reducing the spatial considerations to one dimension. It is the main purpose of this paper to carry out the analysis of a diffusing single species along such a transect.

Such models have been considered in [**2, 3**] where the growth law of the species is logistic without a stability analysis and in [**5, 6, 7**] where the environment is divided into a finite number of patches, each with constant diffusion and constant carrying capacities. Here we continuously vary the diffusion and the carrying capacity spatially.

Received by the editors on September 2, 1992.
Research of the first author carried out while visiting the University of Alberta on a Government of Canada scholarship.
Research of the second author partially supported by the Natural Sciences and Engineering Research Council of Canada, grant no. NSERC A4823.