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AUTOMORPHISMS AND DERIVATIONS OF DIFFERENTIAL EQUATIONS AND ALGEBRAS

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Dedicated to Paul Waltman on the occasion of his 60th birthday

1. Introduction and main results. We consider autonomous differential equations

$$\dot{X} = F(X) \tag{1}$$

in $\mathbf{R}^n$ where $F : \mathbf{R}^n \to \mathbf{R}^n$ is smooth and $\dot{X} = (dX/dt)$. An automorphism of F is an invertible linear transformation $\varphi : \mathbf{R}^n \to \mathbf{R}^n$ satisfying

$$F(\varphi X) = \varphi F(X) \tag{2}$$

for all $X \in \mathbf{R}^n$. The set $\operatorname{Aut} F$ of all automorphisms of F is a closed (Lie) subgroup of $GL(n, \mathbf{R})$. Equivalently, one can define $\operatorname{Aut} F$ to be the largest closed subgroup of $GL(n, \mathbf{R})$ relative to which F is $(\operatorname{Aut} F)$-equivariant.

If $\phi_t(X)$ denotes the flow associated with equation (1), then for each $\varphi \in \operatorname{Aut} F$ we have

$$\phi_t \circ \varphi = \varphi \circ \phi_t. \tag{3}$$

Conversely, any invertible linear transformation satisfying (3) is an automorphism of F (see [**5**, Lemma 5.2]).

A derivation of F is a linear transformation $D : \mathbf{R}^n \to \mathbf{R}^n$ satisfying

$$DF(X) = F'(X) \cdot DX \tag{4}$$

for all $X \in \mathbf{R}^n$; here $F'(X) \cdot Y = (dF(X + sY)/ds)|_{s=0}$. The set $\operatorname{Der} F$ of all derivations of F is a Lie subalgebra of $gl(n, \mathbf{R})$. If $D \in \operatorname{Der} F$,

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