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TRAVELLING WAVE SOLUTIONS OF REACTION-DIFFUSION MODELS WITH DENSITY-DEPENDENT DIFFUSION

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Dedicated to Paul Waltman on the occasion of his 60th birthday

1. Introduction. In a recent paper [**3**], we gave a technique for approximating travelling wave solutions $u(x-ct)$ of reaction-diffusion equations for large wave speed c. The approximation is asymptotic in the sense that it converges uniformly to the exact solution on the set of real numbers as $c \to \infty$.

Below we extend this technique to the case where the diffusion term is a function of u. Such equations are frequently used in mathematical biology to model dispersal of an animal population when there is increased diffusion due to population pressure (see [**1, 4, 5**]).

We will also show how to construct and verify "higher order" approximations. These approximations will improve the earlier ones when c is sufficiently large.

Specifically, consider the equation

$$u_t = D(u)_{xx} - f(u), \tag{1}$$

where D and f are smooth functions satisfying $D(0) = 0$, $D(u) > 0$, $D'(u) > 0$, $f(0) = f(1) = 0$, and $f(u) < 0$ for $0 < u < 1$. We seek travelling wave solutions of the form $u(z) = u(x-ct)$ and find from (1) that u satisfies

$$-c\frac{du}{dz} = \frac{d^2}{dz^2}D(u) - f(u).$$

Now let $\varepsilon = c^{-2}$, $w = \varepsilon^{1/2}z$. Then equation (1) transforms to

$$\varepsilon\frac{d^2}{dw^2}D(u) + \frac{du}{dw} - f(u) = 0. \tag{2}$$

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