

**ON THE EXISTENCE OF THE
CARATHÉODORY SOLUTIONS FOR
SOME BOUNDARY VALUE PROBLEMS**

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Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. In the paper we give some sufficient conditions for the existence of Carathéodory solutions to the Darboux-Goursat boundary value problem and the Dirichlet problem.

0. Introduction. The Darboux-Goursat and Dirichlet boundary value problems are, in general, considered in the Sobolev-type functional spaces where the differentiation of functions is understood in the generalized sense (cf., e.g., [7, 6]).

In the present paper we give some sufficient conditions for the existence of Carathéodory solutions to the Darboux-Goursat problem in the space of absolutely continuous functions in R^2 . By a Carathéodory solution we mean here an absolutely continuous function which possesses the partial derivatives in the classical sense and satisfies the equation under consideration almost everywhere.

Making use of the properties of absolutely continuous functions, one can prove some generalization of the Du Bois-Reymond lemma (cf. [3]) for functions of several variables. The formulation of this lemma and an application to the Dirichlet-type boundary problems are given in the final part of the paper.

1. Absolutely continuous functions of several variables. Denote by P^k an interval in the space R^k , $k \geq 1$, of the form

$$P^k = \{x \in R^k; 0 \leq x^i \leq 1, i = 1, 2, \dots, k\}.$$

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