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# SINGULAR PERTURBATIONS IN VISCOELASTICITY

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Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. We study the singular perturbation for a class of partial integro-differential equations in viscoelasticity of the form

$$\text{(a)} \qquad \begin{aligned}\rho u_{tt}^{\rho}(t,x) = E u_{xx}^{\rho}(t,x) + \int_{-\infty}^{t} a(t-s) u_{xx}^{\rho}(s,x)\,ds \\ + \rho g(t,x) + f(x),\end{aligned}$$

when the density $\rho$ of the material goes to zero. We will prove that when $\rho \to 0$ the solutions of the dynamical systems (a) (with $\rho > 0$) approach the solution of the steady state obtained from equation (a) with $\rho = 0$. The technique of energy estimates is used. A similar result is also obtained for a nonlinear equation of the form

$$\rho u_{tt}^{\rho}(t,x) = \phi(u_x^{\rho}(t,x))_x + \int_{-\infty}^{t} a(t-s)\phi(u_x^{\rho}(s,x))_x\,ds + \rho g(t,x).$$

**1. Introduction.** Consider the following model in viscoelasticity in the one-dimensional case on the real line, (see [**4, 10**]),

$$\begin{aligned}\rho u_{tt}^{\rho}(t,x) &= E u_{xx}^{\rho}(t,x) + \int_{-\infty}^{t} a(t-s) u_{xx}^{\rho}(s,x)\,ds \\ &\qquad + \rho g(t,x) + f(x), \qquad (t,x) \in \mathbf{R}^+ \times [0,1], \\ u^{\rho}(t,0) &= u^{\rho}(t,1) = 0, \qquad t \in \mathbf{R}^+, \\ u^{\rho}(t,x) &= v^{\rho}(t,x), \qquad (t,x) \in \mathbf{R}^- \times [0,1].\end{aligned}$$

Here $u$ is the displacement, $\rho g$ is the body force, $f$ is the external force, $\rho > 0$ is the density of the material and $\mathbf{R}^+ = [0,\infty)$, $\mathbf{R}^- = (-\infty, 0]$.

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