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COUPLED ELASTIC AND VISCOELASTIC RODS

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Dedicated to Paul Waltman on the occasion of his 60th birthday

ABSTRACT. We examine the spectrum for equations for longitudinal vibrations in coupled elastic and viscoelastic rods. A fractional derivative model is used for the viscoelastic rod. We show that, except in an exceptional case, the spectrum asymptotically decomposes into two sets corresponding to the elastic and viscoelastic parts, respectively. Thus, the model can be said to decouple.

1. Preliminaries. In this paper we formulate a linear model for coupled elastic and viscoelastic rods and examine the spectrum of this model. The model is the standard one for a one-dimensional elastic rod, while we employ a fractional derivative type model for the viscoelastic rod. Fractional derivative models for viscoelastic materials have been used frequently in recent years in a variety of studies. A representative list of references includes [**1**–**6**]. We note particularly the paper [**4**] by Desch and Miller where they examine equations with singular kernels. They show that a fractional derivative model for a viscoelastic rod can be written in the more usual form of a partial differential integral equation with a singular kernel. In particular, there are two types of kernels possible. One which we refer to as "strong" while the other is "weak." This, of course, refers to the type of singularity of the kernel at the origin. The treatment of each of these types has similar motivation, but the details are sufficiently different to require that they be considered separately. In each case we will treat the problem as a perturbation problem and show that a characteristic equation has solutions in a prescribed region with the aid of Rouché's theorem.

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