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# ON CONVERGENT $(0,3)$ INTERPOLATION PROCESSES

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Dedicated to Professor A. Sharma on his 70th birthday

ABSTRACT. It is proved that the "pure" and "modified" (0,3) interpolation operators based on the roots of the integral of the Legendre polynomials uniformly converge for all continuous functions. Until now, no such algebraic Birkhoff interpolation was known. Error estimates in terms of moduli of continuity, as well as the optimal order of uniform convergence are determined.

**Introduction.** As the pioneering work of J. Balázs and P. Turán [**2**] has shown, the (0,2) interpolation (i.e., when function values and zero second derivatives are prescribed) based on the roots

$$-1 = x_1 < x_2 < \cdots < x_n = 1 \tag{1}$$

of the polynomial

$$\pi_n(x) = (1-x^2)P'_{n-1}(x), \tag{2}$$

converges uniformly for *some* continuous functions in the interval $[-1,1]$. (Here $P_n(x)$ is the Legendre polynomial of degree $n$ normed such that $P_n(1)=1$.) The condition of convergence was later improved by G. Freud [**4**] and H. Gonska [**5**], but as P. Vértesi [**8**] has shown, the procedure is not uniformly convergent for *all* continuous functions, the reason being that the Lebesgue constant of this type of interpolation is of order exactly $O(n)$. The situation is similar for other classical systems of nodes, and the conjecture is that whenever the (0,2) interpolating polynomials exist, they always diverge for some properly chosen continuous function.

Thus, looking for Birkhoff interpolation procedures that are uniformly convergent for all continuous functions, one may turn to higher order

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