ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 24, Number 2, Spring 1994

WEAK COMPACTNESS IN THE SPACE OF VECTOR-VALUED MEASURES OF BOUNDED VARIATION

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ABSTRACT. Let X be a Banach space and (Ω, Σ) a measure space. A characterization of relatively weak compact subset of the space of X-valued countably additive vector measures of bounded variation defined on Σ is given.

1. Introduction. Let X be a Banach space, (Ω, Σ) a measure space. We denote by $M(\Omega, X)$ the space of X-valued-countably additive measure on (Ω, Σ) of bounded variation.

Recently Ülger [**6**], Diestel, Ruess and Schachermayer [**2**] gave a characterization of weakly compact subsets of $L^1(X)$. The only known characterization of weakly compact subsets of $M(\Omega, X)$ is given by the following theorem:

Theorem A (Bartle-Dunford-Schwartz) [**3**, p. 105]. *Suppose X and X^* have the Radon-Nikodym property* (RNP). *A subset K of $M(\Omega, X)$ is relatively weakly compact if and only if*

(i) *K is bounded,*

(ii) *K is uniformly countably additive,*

(iii) *For each $A \in \Sigma$, the set $\{G(A); G \in K\}$ is a relatively weakly compact subset of X.*

It turns out that one can show, using similar methods as in [**4**], that if (i), (ii) and (iii) are to characterize relatively weakly compact subsets of $M(\Omega, X)$, then X and X^* must have the Radon-Nikodym property.

The use of the Radon-Nikodym derivative was essential in the proof of Theorem A to reduce the study of weakly relative compact subsets of

Received by the editors on January 2, 1992.
AMS (MOS) *Subject Classification* (1980). Primary 46E40, 46G10, Secondary 28B05, 28B20.
This work will constitute a portion of the Ph.D. thesis of the first named author at the University of Missouri-Columbia.