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ISOTOPIC HOMEOMORPHISMS AND NIELSEN FIXED POINT THEORY

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ABSTRACT. Given a topological manifold, M, and an embedding $h : M \to M$ we define a certain class of balls in M called isotopy-standard-for h. The existence of such balls is established and also the following fixed point removability criterion is given: if B is isotopy-standard-for h, and $h|B$ has index zero, then there is an isotopy with support on B taking h to a map which has no fixed points in B.

1. Introduction. For a large class of compact topological spaces, Nielsen theory provides a way to estimate the number of fixed points for any given self-map. If X is a compact polyhedron and $f : X \to X$ a self-map, then the *Nielsen number*, $N(f)$, has the two important properties given below:

(1) $N(f)$ is a lower bound for the number of fixed points of $f : X \to X$,

(2) $N(f)$ is a homotopy invariant (i.e., homotopic maps have the same Nielsen number).

The definition of $N(f)$ and some of its properties can be found in either [**2**] or [**5**]. Loosely, it gives the number of fixed point classes of f which have nonzero index.

A very natural question–and one of general interest–concerns the realizability of the Nielsen number as a lower bound. Specifically, given $f : X \to X$ does there exist a map g homotopic to f having exactly $N(f)$ fixed points? The first affirmative results were due to Nielsen (maps on the torus) and Wecken (maps on PL manifolds of dimension greater than two). Results by a number of authors have since culminated in the following theorem due to Jiang [**6**].

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