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A CHARACTERIZATION OF SOLUTIONS TO A PERTURBED LAPLACE EQUATION II

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1. Introduction. This report is a sequel to work done by P. Staples and the first author in [**12**]. Both papers concern the elliptic partial differential equation

$$\operatorname{div}(\sigma \operatorname{grad} u) = 0, \tag{1.1}$$

which is the model equation for a number of physical situations, e.g., steady state temperature distribution without heat sources where σ is the coefficient of heat conduction of the medium; magnetic potential with σ the magnetic permeability of the medium; the potential of the electric field of a steady current where σ is the conductivity of the medium [**2**, p. 387].

We are interested in finding a representation for the general solution to (1.1) in the case of variable σ, and where σ is *not* required to be *real analytic* in its variables. As in [**12**] we consider only the two dimensional case, where in polar coordinates (r, θ) (1.1) becomes

$$\sigma \Delta u + \sigma_r u_r + \frac{1}{r^2}\sigma_\theta u_\theta = 0. \tag{1.2}$$

Here Δ is the Laplace operator and subscripts denote partial derivatives. In [**12**] σ was assumed to depend on r only. The method of separation of variables was then used to find an eigen-function expansion for the general solution to (1.2) in the unit disk, where classical stability theory for ordinary differential equations was invoked to handle the ensuing r-equation.

In this paper we take the next obvious step and assume $\sigma(r, \theta) = \sigma_1(r)\sigma_2(\theta)$, in order to examine the θ-dependence in the method of separation of variables. An eigenfunction expansion (see (2.20) and (4.4)) for the general solution to (1.2) on the unit disk in this case

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