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SMOOTH POINTS OF VECTOR VALUED FUNCTION SPACES

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ABSTRACT. If E is a Banach space, then an element $x \in E$, $||x|| = 1$ is called smooth if there is a unique $x^* \in E^*$, $||x^*|| = 1$ such that $\langle x^*, x\rangle = 1$. The object of this paper is to characterize the smooth points of $L^p(I, X)$, $l^p(X)$, $1 \leq p < \infty$, where X is some Banach space. Some other related results are presented.

0. Introduction. Let E be a Banach space and $B_1(E)$ the unit ball of E. A point $x \in B_1(E)$ is called a smooth point if there is a unique point $x^* \in E^*$, the dual of E, such that $||x^*|| = 1$ and $\langle x^*, x\rangle = 1$.

In [**4**] Holub studied the smooth points of the unit ball of compact operator and the nuclear operators on a Hilbert space. In [**1**] the authors characterized the smooth points of the unit ball of compact operators and the bounded operator of l^p. Singer [**6**] characterized the smooth points of the unit ball of $C(I, X)$, the space of continuous functions with values in the Banach space X. The object of this paper is to study the smooth points of the unit ball of $L^p(I, X)$ and $l^p(I, X)$ where I is a finite measure space and $1 \leq p < \infty$. Further, we show that $B_1(l^p)$, $0 < p < 1$, has no smooth points. Examples of smooth points of the nuclear operators on l^p, $1 \leq p$, are presented.

Throughout this paper, if X is a Banach space, X^* denotes the dual of X. The projective tensor product of l^p with l^q is denoted by $l^p \hat{\otimes} l^q$. The nuclear operators from l^p to l^q is denoted by $N(l^p, l^q)$, and for $T \in N(l^p, l^q)$ we let $||T||_1$ denote the nuclear norm of T. For an element x in l^p, we write $\operatorname{supp}(x) = \{n : x(n) \neq 0\}$, and $\delta_i = (0, \dots, 0, 1, 0, \dots)$ where 1 appears in the ith-coordinate. We refer to [**3**] for basics of nuclear operators on Banach spaces and for the basic theory of Bochner integrable functions.

1. Smooth points in $L^p(I, X)$. Let I be the unit interval $[0, 1]$

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