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A QUASILINEAR TWO POINT BOUNDARY VALUE PROBLEM

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1. Introduction. With $Du = du/dx$ and $\Omega = (0,1)$ the open unit interval, let

$$Lu = -D[(a_1 + a_2)Du] \tag{1.1}$$

In this representation of $L, a_1(x)$ and $a_2(x)$ will both satisfy $(a-1)$ and $(a-2)$ where with $W^{1,\infty}(\Omega)$ the usual Sobolev space of functions with bounded derivatives in Ω, these two conditions are given as follows:

(a-1) $\quad a(x)\quad$ is a real-valued function in $C(\bar{\Omega}) \cap C^1(\Omega) \cap W^{1,\infty}(\Omega)$;

(a-2) $$\exists \varepsilon_0 > 0 \quad \text{s.t. } a(x) \geq \varepsilon_0 \qquad \forall\, x \in \bar{\Omega}.$$

To L, we associate the quasilinear differential operator

$$\begin{aligned} Qu = -D\left[\sum_{j=1}^{2} a_j(x)\sigma_{ij}(u)Du\right] &+ \sigma_{21}(u)b_1(x,u)[Du]^+ \\ &+ \sigma_{22}(u)b_2(x,u)[Du]^- \end{aligned} \tag{1.2}$$

where

$$\sigma_{ij} : W_0^{1,2}(\Omega) \to \mathbf{R} \quad \text{with } \sigma_{ij} \text{ continuous in the strong } W_0^{1,2}\text{-topology for } i,j = 1,2, \text{ and} \tag{1.3}$$

$$b_j(x,s) \in C[\bar{\Omega} \times \mathbf{R}] \quad \text{for } j = 1,2. \tag{1.4}$$

Also,

$$[Du(x)]^+ = \max[Du(x), 0], \qquad [Du(x)]^- = \max[-Du(x), 0].$$

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