ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 24, Number 3, Summer 1994

POINT-TO-LINE DISTANCES IN THE PLANE OF A TRIANGLE

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ABSTRACT. Many inequalities involving notable points and lines in the plane of a triangle ABC are presented. Most are new; however, they are only conjectured, not proved. Each inequality was detected by computer and then confirmed for 10,740 triangles, selected in such a way that it will be remarkable if any of the conjectures should eventually prove to be false. Ninety-one specific notable points are considered, along with two notable lines, namely, the Euler line, $\mathcal{E}$, and the line perpendicular to $\mathcal{E}$ that passes through the orthocenter of ABC. Typical of the hundreds of inequalities is that the distance from the incenter to $\mathcal{E}$ never exceeds the distance from the symmedian point to $\mathcal{E}$.

1. Introduction. "Computers can solve mathematical problems," writes David Gale [**3**]. "They can also pose them and now, it seems, they may be capable of killing off whole branches of the subject." Gale refers specifically to Euclidean geometry, and even more specifically to a list $\mathcal{L}$ of notable points in the plane of a triangle which exhibit many newly-discovered-by-computer collinearities and other properties. The computer detects *all* cases of collinearity among points in $\mathcal{L}$, so that no further collinear subsets of $\mathcal{L}$ will remain to be found in the future. In that sense, the computer may be capable of killing off a branch, or pruning a twig, of mathematics.

In the same killing spirit we now use the computer to investigate distances among well-known notable points and lines of a triangle. An example of such an inequality is

$$\text{(distance from incenter to Euler line)} \\ \leq \text{(distance from symmedian point to Euler line)}.$$

There appear to be an astonishing number of new inequalities of this sort. In order to list them, it is helpful to use an indexing of notable points, or *centers*, as introduced in [**5**]. Here we consider a total of 91

Received by the editors on January 18, 1993.