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CONFORMAL INVARIANTS FOR CURVES AND SURFACES IN THREE DIMENSIONAL SPACE FORMS

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ABSTRACT. Conformal invariants of submanifolds have been known for a hundred years. And yet many recent papers seem to ignore the major works of yesteryear and the complete list of invariants that they defined. This present paper aims to provide a readable account of the existence of conformal invariants in their most natural setting, that of curves and surfaces in 3-space.

1. Introduction. In the same way that curves and surfaces possess invariants under the group of isometries (curvature, torsion, principal curvatures, etc.), it has been known since the beginning of this century that there are other functions which are invariant under the larger group of conformal transformations; hence there is a *conformal curvature*, a *conformal torsion*, etc. Now, conformal invariants have received a revival of interest in recent years [**8, 46, 22, 18, 19, 17**], in part because of Willmore's conjecture [**47**], and the associated conformal invariant known as the Willmore integrand (see [**31**]). Nevertheless, despite this interest, there have been very few references in the literature to the complete list of invariants and to the papers which established them (important exceptions are [**20, 33, 35, 36**]. See also [**45**]). The purpose of this present paper is to provide what we hope will be a timely review of these invariants.

To be specific, we are concerned in this paper with conformal geometry in the following sense. Let N denote a 3-dimensional Riemannian manifold with metric g of constant sectional curvature (which means that locally it is elliptic, hyperbolic or Euclidean space). We study those properties of N which remain invariant under conformal change of metric, that is under replacement of g by ρg, where ρ is a positive function on N and ρg still has constant sectional curvature, but possibly with a different constant. For example we can take N to be the

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