

ALGEBRAIC DYNAMICS OF POLYNOMIAL MAPS ON THE ALGEBRAIC CLOSURE OF A FINITE FIELD, II

ANJULA BATRA AND PATRICK MORTON

ABSTRACT. We study the dynamics of polynomial maps on the algebraic closure of the finite field \mathbf{F}_q by associated to a polynomial $\sigma(x)$ in $\mathbf{F}_q[x]$ a graph G_σ on the irreducible polynomials over \mathbf{F}_q which reflects the algebraic properties of the mapping $\alpha \rightarrow \sigma(\alpha)$. For additive polynomials σ we show that many of the connected components of G_σ are isomorphic to the connected component of x , and we determine the structure of all of the connected components of G_σ over \mathbf{F}_p explicitly when $\sigma(x) = x^p \pm x$ and p is prime. We also describe the connection between the graph G_σ for $\sigma(x) = x^p - x$ and Artin-Schreier theory.

1. Introduction. In Part I of this paper we have defined a graph G_σ on the monic, irreducible polynomials over a finite field \mathbf{F}_q which reflects the dynamics of the mapping $a \rightarrow \sigma(a)$ on the algebraic closure of \mathbf{F}_q , where $\sigma(x)$ is a nonconstant polynomial with coefficients in \mathbf{F}_q (the same definitions work for an arbitrary field). In that paper we also proved a number of theorems about the cycles in the graph G_σ , and gave special consideration to the polynomials of the form $\sigma(x) = x^q + ax$.

In this part of the paper we will first investigate the structure of the connected components of this graph for general additive (separable) polynomials and then give more detailed results for two special families of polynomials, the maps $\sigma(x) = x^p \pm x$, considered over the prime field \mathbf{F}_p . When we want to emphasize the ground field κ , we use the notation $G_\sigma(\kappa)$. The connected component of a polynomial f in G_σ will be denoted by $C_\sigma(f)$ or by $C_\sigma(f; \kappa)$ if the field κ needs to be emphasized.

We recall that for two irreducible polynomials f and g over \mathbf{F}_q , the edge $g \rightarrow f$ is in the graph G_σ if and only if the map $\sigma(x)$ takes a root of g to a root of f . In part 1 we show that $g \rightarrow f$ for a unique

Received by the editors on May 13, 1993, and in revised form on October 12, 1993.

1991 AMS *Mathematics Subject Classification.* 12E20, 5C20.

The second author was supported by a Brachman-Hoffman grant from Wellesley College, and both authors were supported by NSF Grant DMS-9200575, during the period in which this article was written.

Copyright ©1994 Rocky Mountain Mathematics Consortium