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ABELIAN GROUPS WITH SEMI-SIMPLE ARTINIAN QUASI-ENDOMORPHISM RINGS

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1. Introduction. In 1937, Baer published a paper which can still be considered as probably the most important contribution to the theory of torsion-free abelian groups [**8**]. In it, he gave a complete set of numerical invariants for the subgroups of the rational numbers, $\mathbf{Q}$, and their direct sums. Particular attention was given to homogeneous completely decomposable groups, which are groups of the form $\oplus_I A$ for some subgroup A of $\mathbf{Q}$. Baer's investigations resulted in a series of splitting properties for these groups, which have become important tools in the discussion of torsion-free abelian groups of finite rank [**10**, Section 86].

Because of this, the question arose whether it is possible to replace the subgroup A of $\mathbf{Q}$ in the definition of a homogeneous completely decomposable group by a more general group without losing the previously mentioned splitting properties. Although it soon became apparent that some restrictions on A are needed, it was not until 1975 that Arnold and Lady showed that the most natural way to introduce these restrictions is in terms of the endomorphism ring, $E(A)$ of A.

Following their approach, we consider a torsion-free abelian group A and call a group P *A-projective of finite A-rank* if P is a direct summand of A^n for some $n < \omega$. If G is an abelian group, then $S_A(G) = \langle f(A) \mid f \in \operatorname{Hom}(A, G)\rangle$ is the *A-socle of G*, while $R_A(G) = \cap\{\ker f \mid f \in \operatorname{Hom}(G, A)\}$ denotes the *A-radical of G*.

[**6,2**] and [**3**] were mainly concerned with the splitting and quasi-splitting of exact sequences of the form $0 \to P \xrightarrow{\alpha} G \xrightarrow{\beta} B \to 0$ and $0 \to B \xrightarrow{\alpha} G \xrightarrow{\beta} P \to 0$ in which P is a quasi-summand of an A-projective group of finite A-rank. We say that A has the *radical-splitting property* if every such sequence $0 \to P \xrightarrow{\alpha} G$ with $\alpha(P) \cap R_A(G) = 0$ quasi-splits, while A has the *finite quasi-Baer-*

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