

ISOMETRIES OF MUSIELAK-ORLICZ SPACES EQUIPPED WITH THE ORLICZ NORM

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ABSTRACT. In [2, 4] a characterization of the group of surjective isometries of the complex Musielak-Orlicz space L_Φ equipped with the Luxemburg norm was given. Here it is shown that the same characterization also remains valid for the group of isometries of L_Φ endowed with the Orlicz norm.

1. Introduction. In this paper we will investigate surjective isometries of complex Musielak-Orlicz spaces equipped with the Orlicz norm. The main conclusion is that the groups of isometries with respect to Orlicz or Luxemburg norms are the same. The analogous fact for Orlicz spaces has already been observed in [13], however without any proof. Thus the results presented here provide a justification also for the mentioned remark in [13].

As a first step we will give a characterization of support functionals of elements in E_Φ . Then, by means of that characterization, we will show that any Hermitian operator H of L_Φ equipped with the Orlicz norm has a diagonal form, that is, $H(f) = h \cdot f$ for some real bounded function h . This description of Hermitian operators appears to be the same as for the Luxemburg norm. Hence we conclude that any surjective isometry preserves disjointness of the supports of functions. Finally, applying the already known criterion of isometries of L_Φ endowed with the Luxemburg norm, we will prove that the criterion is the same if L_Φ is considered with the Orlicz norm.

Although the paper is a continuation of [2] and [4] and will often refer to the results of those papers, for the convenience of the reader, we recall some definitions and facts on isometries, Hermitian operators and Musielak-Orlicz spaces.

In this paper only surjective isometries will be investigated. Thus we will say a transformation U of a Banach space $(X, \|\cdot\|)$ is an *isometry* if it is linear, surjective and if it preserves the norm, that is, $\|Ux\| = \|x\|$ for all $x \in X$.

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