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# ON THE DIVISIBILITY OF $h^+$ BY THE PRIME 3

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**Introduction.** Let $l$ and $p$ be primes such that $p = 2l + 1$. In the paper [**1**] it is proved that if 2 is a primitive root modulo $l$ then 2 does not divide class number of real cyclotomic field $\mathbf{Q}(\zeta_p + \zeta_p^{-1})$. In the paper [**3**] it is proved that the same result holds for arbitrary prime $q$ which is primitive root modulo $l$. In [**2**] it is shown that, provided the order of 2 modulo $l$ is $(l-1)/2$ and 2 is prime in the real subfield of $\mathbf{Q}(\zeta_l)$, then 2 does not divide the class number of real cyclotomic field $\mathbf{Q}(\zeta_p + \zeta_p^{-1})$.

The aim of this paper is to prove the same result for the prime 3.

The following theorem holds.

**Theorem.** *Let $l$ and $p$ be primes such that $l > 3$, $p = 2l + 1$, and the order of 3 modulo $l$ is $(l-1)/2$. Then 3 does not divide the class number $h^+$ of real cyclotomic field $\mathbf{Q}(\zeta_p + \zeta_p^{-1})$.*

*Proof.* Clearly $l \equiv p \equiv 2 \pmod 3$. Since the order of 3 modulo $l$ is $(l-1)/2$ we have $(3/l) = 1$. If $l \equiv 1 \pmod 4$, then

$$1 = (3/l) = (l/3) = (2/3) = -1.$$

Hence $l \equiv 3 \pmod 4$ and it follows that 3 is prime in the real subfield of $\mathbf{Q}(\zeta_l)$.

In [**3**] it is proved if $3|h^+$ then $3|N_{\mathbf{Q}(\zeta_l)/\mathbf{Q}}(\omega)$, where

$$\omega = \sum_{i \equiv 1 \pmod 3} \chi(i),$$

and $\chi$ is the Dirichlet character modulo $p$ defined by $\chi(x) = \zeta_l^{\operatorname{ind} x}$.

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