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REMAINDERS, SINGULAR SETS AND THE CANTOR SET

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ABSTRACT. Let X be a completely regular Hausdorff space which is not locally compact. Characterizations are given for when X has a compactification αX for which $\mathrm{Cl}_{\alpha X}(\alpha X - X)$ is the Cantor set C. This occurs if and only if C is the singular set of a continuous function.

For such spaces, there is also a compactification αX for which $\mathrm{Cl}_{\alpha X}(\alpha X - X)$ is the closed unit interval in case X has a residue which is countable.

1. Introduction. All topological spaces considered here are completely regular and Hausdorff. A remainder of a Hausdorff compactification αX of a space X is the set $\alpha X - X$. Substantial investigation has been devoted to the question of which spaces Y can serve as remainders for a space X. (See [**3, 5, 7** and **8**], for example.) Y. Unlü [**12**] and the present authors [**6**] have characterized when the Cantor set C is a remainder of a locally compact space X. Clearly, C cannot be a remainder of a nonlocally compact space.

In this paper we characterize when, for nonlocally compact X, there is a compactification αX of X for which the closure of $\alpha X - X$ in αX is C. For any X we let $R(X)$ denote the set of all points in X which do not possess a compact neighborhood. Then $\mathrm{Cl}_{\alpha X}(\alpha X - X) = (\alpha X - X) \cup R(X)$. Thus, for spaces satisfying $C = \mathrm{Cl}_{\alpha X}(\alpha X - X)$, it follows that $R(X)$ is a subset of C. When X is almost locally compact, that is, when $X - R(X)$ is dense in X, (see [**10**]), and $L(X)$ is the locally compact part of X, we observe that then αX is also a compactification of $L(X)$ for which $\alpha X - L(X) = C$ so that all compact metric spaces are remainders of $L(X)$ (see [**6**]).

If Y is compact and f is a continuous mapping from X into Y, the singular set of f is the set of all points p in Y for which $\mathrm{Cl}_X f^{-1}(N_p)$

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