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RIESZ DECOMPOSITION IN INDUCTIVE LIMIT C^*-ALGEBRAS

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ABSTRACT. As recently proved by Zhang, the projections in any C^*-algebra of real rank zero enjoy the Riesz decomposition property. Here the Riesz decomposition property is obtained for projections in several types of C^*-algebras with positive real rank, including the inductive limits with slow dimension growth introduced by Blackadar, Dădărlat, and Rørdam. Waiving the dimension restrictions, weaker forms of the Riesz decomposition property are established for general inductive limits of finite direct products of homogeneous C^*-algebras.

1. Introduction and background. The Riesz decomposition property (see below) for projections in C^*-algebras of real rank zero was established by Zhang [**32**, 1.1] and used as a key tool in his investigation of the structure of such algebras and of their multiplier and corona algebras—see, e.g., [**32**, 2.2, 2.3], [**33**, 1.2], [**34**, 1.1]. More recently, Zhang's result has been used by Elliott as one of the key ingredients in his classification of certain C^*-algebras of real rank zero [**10**].

Our aim here is to show that Riesz decomposition is more widespread than Zhang's theorem indicates. We prove it for a large class of C^*-algebras with positive real rank, including all simple inductive limits with slow dimension growth as in [**3**], as well as nonsimple inductive limits satisfying a related form of slow dimension growth. Under somewhat relaxed hypotheses, we obtain weaker forms of Riesz decomposition, sufficient for instance to prove that for any approximately semi-homogeneous C^*-algebra A (see Section 3), the partially ordered abelian group $K_0(A) \otimes \mathbf{Q}$ is a Riesz group.

Our work on inductive limits with slow dimension growth relies on extensions to arbitrary compact Hausdorff spaces of standard cancellation theorems for vector bundles over finite CW-complexes. Various

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