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ON EQUIVALENT CHARACTERIZATIONS OF WEAKLY COMPACTLY GENERATED BANACH SPACES

M. FABIAN AND J.H.M. WHITFIELD

1. Introduction. Let us consider the following nice

Theorem. *For a Banach space V the following assertions are equivalent*

(a) *V is weakly compactly generated* (w.c.g.),

(b) *V is* GSG *and simultaneously a Vašák* (*i.e., weakly K-countable determined*) *space, and*

(c) *V is* GSG *and moreover (V^*, w^*) continuously injects into $\Sigma(\Gamma)$ for some set Γ.*

Recall that it has, according to [**18**, Theorem [S8], **20** and 14, proof of Theorem A, Proposition 4.1], an even nicer, topological counterpart

Theorem′. *The following assertions are equivalent*

(α) *K is an Eberlein compact,*

(β) *K is simultaneously a Radon Nikodým compact and a Gul'ko compact, and*

(γ) *K is simultaneously a Radon Nikodým compact and a Corson compact.*

In the theorem, the implications (a) $\rightarrow$ (b) $\rightarrow$ (c) are not quite new. In fact, according to the interpolation theorem [**12**, p. 163], every w.c.g. space contains a dense continuous image of a reflexive space and hence is GSG. An observation that every w.c.g. space is Vašák is due to Talagrand [**19**]. Finally, the fact that the dual of a Vašák space endowed with the weak* topology continuously injects into $\Sigma(\Gamma)$ is due

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