ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 24, Number 4, Fall 1994

AN INTERPOLATION SCHEME WITH RADIAL BASIS IN SOBOLEV SPACES $H^s(R^n)$

JULIO G. DIX AND ROBERT D. OGDEN

ABSTRACT. This article concerns a global interpolation scheme that uses the radial basis $\exp(-|x-x_i|)P(|x-x_i|)$, where $P(\cdot)$ is a polynomial. These basis functions belong to a Sobolev space $H^s(R^n)$ and correspond to evaluations of reproducing kernels at data points. To balance the visual smoothness of interpolating graphs and the condition number of a resulting linear system, we use a distance scaling factor that depends on the data. We find a rate of convergence for approximations and show results of numerical experiments.

0. Introduction. The interpolation problem that we are interested in consists of finding a function $\tilde{f}$ that assumes given real values at given points $\{x_i\}$ of R^n. This problem has many applications in real life. The intended use of interpolants determines the properties that $\tilde{f}$ should have. For instance, it should have continuous derivatives, or should be easily computable, or should be continuously dependent on the data, etc. A summary of desirable properties of interpolants is found in Grosse [**12**]. A list of interpolation methods is found in Afeld [**1**], and a comparison of methods is found in Franke [**6**].

An interpolation scheme can be global or local. It is global if the interpolant at any given point depends on all the data and local if it depends only on data "near" the given point. Each type of interpolation is suitable for some applications. But no one scheme is best for all applications. The scheme here is global but its basis can be truncated to have a local character.

An interpolation method is called radial if its basis functions depend on x only through the distance to data points, $|x-x_i|$. The general form of radial interpolants is

$$\tilde{f}(x)=\sum_i w_i\phi_i(|x-x_i|), \tag{0.1}$$

Received by the editors on February 11, 1992, and in revised form on June 28, 1993.
1980 *Mathematics Subject Classification.* 41A05, 41A63, 65D05, 65D15.
Key words. Interpolation, scattered data, radial functions, reproducing kernel.