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STRICTLY CYCLIC VECTORS FOR INDUCED REPRESENTATIONS OF LOCALLY COMPACT GROUPS

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ABSTRACT. An induced representation of a locally compact group has a strictly cyclic vector only if the coset space is finite. A nonzero subrepresentation of a representation induced from a compact or normal subgroup has a strictly cyclic vector only if the coset space is compact.

Preliminaries. Throughout G is a separable locally compact group and H is a closed subgroup. Let ν be left Haar measure on G and assume that there exists an invariant measure μ on G/H, the left cosets. Let π be a continuous unitary representation of H on a Hilbert space $\mathcal{H}(\pi)$ and ϕ a function from G to $\mathcal{H}(\pi)$ such that $\phi(xh) = \pi(h^{-1})\phi(x)$ for all $x \in G$ and all $h \in H$. Since π is unitary, the function $x \to ||\phi(x)||$ is constant on the left cosets of H. Therefore the space of weakly measurable functions $\phi : G \to \mathcal{H}(\pi)$ satisfying

i) $\phi(xh) = \pi(h^{-1})\phi(x)$ for $x \in G$ and $h \in H$, and

ii) $\int_{G/H} ||\phi(x)||^2 \, d\mu < \infty$

is a Hilbert space under the inner product $\langle \phi, \gamma \rangle = \int_{G/H} \langle \phi(x), \gamma(x) \rangle \, d\mu$. The induced representation π^G of G on this space, denoted by $\mathcal{H}(\pi^G)$, is defined by $\pi^G(s)\phi(x) = \phi(s^{-1}x)$. It follows that π^G is a continuous unitary representation of G, see [**4**].

Main results. For $f \in L_1(G)$ define the operator $\pi^G(f)$ on $\mathcal{H}(\pi^G)$ by $\int_G f(x)\pi^G(x) \, d\nu$, where this integral is taken in the weak sense. Then $||\pi^G(f)|| \leq ||f||$. Therefore the map π^G defines a continuous representation of the Banach $*$-algebra $L_1(G)$ on $\mathcal{H}(\pi^G)$. Fix $\phi \in \mathcal{H}(\pi^G)$ and define the map T_ϕ from $L_1(G)$ to $\mathcal{H}(\pi^G)$ by $T_\phi f = \pi^G(f)\phi$. Then $||T_\phi|| \leq ||\phi||$. Let T_ϕ^* denote the adjoint map. Then $T_\phi^* : \mathcal{H}(\pi^G) \to L_\infty(G)$ and $||T_\phi^*|| \leq ||\phi||$.

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