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# THOMAS'S STRUCTURE BUNDLE FOR CONFORMAL, PROJECTIVE AND RELATED STRUCTURES

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**1. Introduction and conventions.** Local twistors for four-dimensional conformal spin manifolds were described by Penrose and others [**8, 19, 21**] as a means of applying twistor theory to curved space-times. Numerous authors later realized that the local twistor bundle is an associated vector bundle (via a spin representation) of the Cartan conformal connection [**18**]. We had been using a generalization of this calculus for some time in connection with various problems in conformal and projective geometry when C. Fefferman and C.R. Graham pointed out to us that these methods go back to T.Y. Thomas ([**27**] for the conformal case and [**26**] for the projective), who discovered the "Cartan connections" a little later, but independently of E. Cartan. Thomas's treatment is somewhat difficult for the modern reader since it predates the idea of a vector bundle. His work however is essentially complete, and thus considerably ahead of its time.

The difference between the Cartan approach and the Thomas approach is that the former works with a principal bundle, whereas the latter takes a certain associated vector bundle as the starting point. Of course, principal bundle methods are very powerful, but on the other hand we, at least, do not at an intuitive level think of (for example) a vector field as a function on a principal bundle. We regard Thomas's calculus as the "intuitive" version of the Cartan connections. It has the added advantage that there is a definition of Thomas's vector bundle which is quite direct, in contrast to the construction of the principal bundle for the Cartan connection.

One can perform Thomas's construction for any structure that has a "Cartan connection." Here we treat conformal and projective structures, and mention paraconformal structures [**1**] briefly. We wish to

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