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EQUIVALENT DYNAMICS FOR A STRUCTURED POPULATION MODEL AND A RELATED FUNCTIONAL DIFFERENTIAL EQUATION

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In [**4**], the author established a relationship between the semiflows and global attractors for a simple structured population model of juvenile versus adult competition and a related functional differential equation (FDE) constructed from an analysis of the model. Building on this earlier work, we show in this paper that, under an additional assumption, the semiflows of the two systems, restricted to their respective global attractors, extend to flows which are topologically equivalent. In other words, the long-term dynamics generated by the hyperbolic system of equations representing the structured population model is faithfully represented by the long-term dynamics of the FDE. In particular, the wealth of theory available for the study of FDEs can be brought to bear on the problem of determining the long-term dynamics for the model. It should be emphasized that, at present, there does not exist a correspondingly well-developed theory for the analysis of structured population models.

Let us emphasize that the main point of this paper is to establish rigorously that in order to study the asymptotic behavior of the model system, it suffices to do the same for the simpler scalar FDE.

Future work will focus on describing the range of possible dynamical behavior for the FDE. In [**3, 5**] it was shown that periodic solutions can occur through Hopf bifurcation from the positive equilibrium.

The system of equations (1) below model the interaction of juveniles and adults of a single species. Adults are viewed as identical in every relevant aspect and their number is denoted by w. Juveniles vary in their level of maturity x between a newborn level $x = 0$ and a pre-adult level $x = 1$. Pre-adult juveniles mature to adults. Juveniles acquire maturity at a rate $dx/dt = P(w)$, that is, as a function of

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