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STABILITY OF RANDOM MATRIX MODELS

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ABSTRACT. Random matrices have been widely studied as neutral models for the stability of large systems and, in particular, ecosystems. However, many ecologists interpret stability in terms of low variability and persistence, not Lyapunov stability studied in matrix theory. Following Harrison [**5**], Hastings [**6**] suggested a close relationship between Lyapunov stability and low variability for random matrix models with additional noise terms. We report on a simulation study confirming this conjecture and its extension to certain products of random matrices.

1. Introduction. "Will a large complex system be stable?" (May [**11**]). This important and intriguing problem has been extensively studied by Gardner and Ashby [**4**], May [**11, 12**], Hastings [**6**], Cohen and Newman [**3**], and Hastings, Juhasz and Schreiber [**7**] using linear models of the form

$$\underline{x}(t+1) = M\underline{x}(t) \tag{1}$$

in which M is a random matrix. Following Wigner [**14**], May [**11**] made the following conjecture.

The May-Wigner stability theorem. *Let M be an $n \times n$ matrix with connectance C (each entry of M is nonzero with probability C, independent of other entries) and root mean square (rms) interaction strength α (the nonzero entries are chosen independently from a symmetric distribution with expected square α^2). Then the spectral radius of M is given by*

$$\rho = \rho(M) = \alpha(nC)^{1/2} \tag{2}$$

asymptotically for large n.

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