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THE CAUCHY FUNCTION FOR nTH ORDER LINEAR DIFFERENCE EQUATIONS

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In this paper we will be concerned with the Cauchy function for the nth order linear difference equation

$$Ly(t) \equiv \sum_{i=0}^{n} p_i(t)y(t+i) = 0 \tag{1}$$

where t is an integer variable, and we assume that

$$p_0(t)p_n(t) \neq 0,$$

for all integers t. We assume the coefficients $p_i(t)$, $0 \leq i \leq n$, are real valued functions defined on the integers. The Cauchy function $K(t,s)$, for each fixed integer s, is defined to be the solution of the initial value problem (1),

$$K(s+k,s) = 0, \qquad 1 \leq k \leq n-1,$$
$$K(s+n,s) = \frac{1}{p_n(s)}.$$

Let a be an integer. Then it is well known that the solution for the initial value problem,

$$\sum_{i=0}^{n} p_i(t)y(t+i) = f(t), \qquad t \geq a$$
$$y(a+k) = 0, \qquad 0 \leq k \leq n-1$$

is given by

$$y(t) = \sum_{s=a}^{t-1} K(t,s)f(s).$$

Assume that we can factor (1) in the form

$$Ly(t) = MNy(t) = 0$$

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