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ON R.A. SMITH'S AUTONOMOUS CONVERGENCE THEOREM

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ABSTRACT. R.A. Smith [19] showed that his higher dimensional generalization of Bendixson's criterion for the nonexistence of periodic solutions to an autonomous differential equation implies that all bounded trajectories tend to an equilibrium. Here it is shown that a similar conclusion can be drawn from a generalized Dulac criterion. These conditions are also shown to have strong implications for the structure of invariant sets.

1. Introduction. Let the map $x \mapsto f(x)$ from an open set $\mathcal{D}$ in $\mathbf{R}^n$ to $\mathbf{R}^n$ be such that each solution $x(t)$ to the differential equation

$$dx/dt = f(x) \tag{1}$$

is uniquely determined by its initial value $x(0) = x_0$ and denote this solution $x(t, x_0)$.

A point $x_0 \in \mathcal{D}$ is *wandering* for (1) if there exists a neighborhood $\mathcal{U}$ of x_0 and $T > 0$ such that $\mathcal{U} \cap x(t,\mathcal{U})$ is empty for all $t \geq T$. Thus, for example, any equilibrium, α limit point or ω limit point is nonwandering.

The closing lemma of Pugh [**15, 16, 17**] shows that if f is C^r and x_0 is a nonwandering point which is not an equilibrium for (1), then there are differential equations arbitrarily C^r-close to (1) which have nonconstant periodic solutions. Suppose now that f satisfies a condition which precludes the existence of periodic solutions to (1). If the condition is sufficiently robust that it is also satisfied by functions which are C^r-close to f, then the closing lemma implies that every nonequilibrium point of (1) is wandering. In particular, every α or ω limit set consists entirely of equilibria; it follows that if the zeros of f are isolated, then

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