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BENDIXSON'S CRITERION FOR AUTONOMOUS SYSTEMS WITH AN INVARIANT LINEAR SUBSPACE

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Dedicated to the memory of Professor G.J. Butler

ABSTRACT. A class of nonlinear autonomous systems ordinary differential equations in $\mathbf{R}^n$ with an invariant linear subspace which includes as examples a wide range of biological and chemical systems is defined and studied. Among other things, criteria precluding the existence of periodic solutions are obtained for such systems using a general method developed in [**4**].

1. Introduction. Let $D \subset \mathbf{R}^n$ be a convex open set and $x \mapsto f(x) \in \mathbf{R}^n$ a C^1 function defined in D. We consider the autonomous system in $\mathbf{R}^n$

$$x' = f(x) \tag{1.1}$$

under the following assumptions:

(H1) The Jacobian matrix $\partial f/\partial x$ of the vector field f of (1.1) can be written as

$$\frac{\partial f}{\partial x}(x) = -\nu I + A(x) \quad \text{for all } x \text{ in } D, \tag{1.2}$$

where ν is a constant and $x \mapsto A(x)$ is an $n \times n$ matrix-valued function.

(H2) There exists a constant matrix B with $\operatorname{rank} B = r$ such that

$$BA(x) = 0 \quad \text{for all } x \text{ in } D. \tag{1.3}$$

We will call a nonlinear system (1.1) satisfying (H1) and (H2) an autonomous system with an invariant linear subspace. Examples of

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