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OSCILLATORY AND ASYMPTOTIC BEHAVIOR OF A DISCRETE LOGISTIC MODEL

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ABSTRACT. We consider the discrete logistic model with or without delay

$$x_{n+1} = \frac{\alpha_n x_n}{1 + \beta_n x_{n-j}}, \qquad n = 0, 1, 2, \dots, j \geq 0$$

where α_n, β_n are positive bounded sequences. A complete discussion on the oscillatory and asymptotic behavior is given for the case that $j = 0$. For the case that $j > 0$, some results on oscillation are also obtained.

1. Introduction. In 1969, Pielou posed the difference equation model (see [**8**])

$$x_{n+1} = \frac{\alpha x_n}{1 + \beta x_{n-j}}, \qquad n = 0, 1, 2, \dots, j \geq 0 \tag{1.0}$$

(where $\alpha > 1$, $\beta > 0$ are constants) as the discrete analog of the delay logistic equation

$$\dot{N}(t) = rN(t)\left[1 - \frac{N(t-\tau)}{p}\right].$$

Recently, Kuruklis and Ladas have obtained oscillation criteria for Equation (1.0) with $j > 0$ and asymptotic stability results for (1.0) with $j = 0, 1$, see [**4**].

However, from the derivation of the model (1.1) we see that α and β are related to the growth rate r and the carrying capacity p as follows:

$$\alpha = e^r \quad \text{and} \quad \beta = (e^r - 1)p,$$

and hence are not constants, and not even periodic in general.

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