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HIERARCHICALLY STRUCTURED BRANCHING POPULATIONS WITH SPATIAL MOTION

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ABSTRACT. We consider hierarchically structured systems of individuals undergoing birth-and-death or branching processes at each level, with spatial motion at the lowest level. The measure-valued continuous diffusion limit process is formulated, and the carrying dimension of the topological support of the underlying random measure is analyzed.

1. Introduction. Systems of particles that undergo simultaneous diffusion and birth-and-death or branching have long been used to model random phenomena in several fields. Early development of the Galton-Watson branching process was stimulated by studies of the spread of wealth and royalty among the gentry in turn-of-the-century England. Years later, such processes were applied to models describing the spread of mutant genes through natural populations, the spread of diseases among susceptible individuals, and the distribution of both neutral and selectively advantageous allelic types in population genetics.

It has, in fact, been the field of population biology which has provided both the motivation and direction for much of the recent work on both interacting and noninteracting branching-diffusing systems. This has certainly been true in the case of the Dawson-Watanabe superprocess, a measure-valued stochastic process that arises as the high-density continuous diffusion limit of an infinite system of noninteracting branching-diffusing individuals or particles. It is even more evident in the case of the measure-valued Fleming-Viot processes, which arise as the diffusion approximations to certain models originally formulated in population genetics.

In this paper we will study certain aspects of hierarchically structured birth-and-death and branching populations with spatial motion.

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