ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 1, Winter 1995

# LIAPUNOV-RAZUMIKHIN FUNCTIONS AND AN INSTABILITY THEOREM FOR AUTONOMOUS FUNCTIONAL DIFFERENTIAL EQUATIONS WITH FINITE DELAY

JOHN R. HADDOCK AND YOUNHEE KO

**1. Introduction and notation.** It is well known that Liapunov's direct method sometimes provides a useful tool in the study of instability of functional differential equations (FDEs). See, for example, [**1**] and [**3, 4**]. However, an obstacle often is encountered when one tries to apply this method; namely, it frequently is difficult—if not impossible—to construct appropriate Liapunov functions or functionals in order to make use of known instability theorems. The purpose of this paper is to provide an instability theorem that eliminates some of the obstacles imposed by this difficulty. In particular, we employ Liapunov-Razumikhin techniques and omega limit set properties in order to present an instability result (Theorem 2.1) for autonomous FDEs with finite delay. An example is given to illustrate that this theorem often is straightforward to apply—when applicable—and can be used to retrieve and extend previously known instability results.

We use the standard notation for finite delay FDEs. Let $|\cdot|$ denote any convenient norm on the real Euclidean space $R^n$ of (column) $n$-vectors. Further, let $r \geq 0$ be given, and let $C = C([-r, 0], R^n)$ with

$$||\phi|| = \max_{-r\leq s\leq 0} |\phi(s)|, \qquad \phi \in C.$$

For $H > 0$, define $C_H \subset C$ by

$$C_H = \{\phi \in C : ||\phi|| < H\}.$$

If $x : [-r, A) \to R^n$ is continuous, $0 < A \leq \infty$, then, for each $t$ in $[0, A)$, $x_t$ in $C$ is defined by

$$x_t(s) = x(t + s), \qquad -r \leq s \leq 0.$$

Received by the editors on September 29, 1992, and in revised form on May 26, 1993.

Both authors were supported in part by the National Science Foundation under Grant DMS 9002431.

Copyright ©1995 Rocky Mountain Mathematics Consortium