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ON THE BEHAVIOR OF SOME EXPLICIT SOLUTIONS OF THE HARMONIC MAPS EQUATION

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1. Introduction and definitions. Harmonic maps of the Minkowski space are the critical points $u : \mathbf{M} \to \mathbf{N}$ of the energy functional

$$\int_{\mathbf{M}} \eta^{\alpha\beta} g_{ij} \frac{\partial u^i}{\partial x^\alpha} \frac{\partial u^j}{\partial x^\beta}\, dx \tag{1.1}$$

where

$$\begin{aligned}&\mathbf{M}(n,1) \text{is the } n+1-\text{dimensional Minkowski space}\\ &\text{with Lorentzian metric } \eta^{\alpha\beta} = (1,-1,\dots,-1) \text{ and local}\\ &\text{coordinates } x^0 = t, x^1, \dots, x^n,\end{aligned} \tag{1.2}$$

$$\begin{aligned}&\mathbf{N}\text{is an } m-\text{dimensional Riemannian manifold with}\\ &\text{local coordinates } (u^1,\dots,u^m) \text{ and metric form}\\ &ds^2 = g_{ij}(u)du^i du^j.\end{aligned} \tag{1.3}$$

The Euler-Lagrange equations describing the critical points of (1.1) are

$$\frac{\partial^2 u^i}{\partial t^2} - \sum_{p=1}^n \frac{\partial^2 u^i}{\partial x^{p2}} + \Gamma^i_{jk}(u)\left\{\frac{\partial u^j}{\partial t}\frac{\partial u^k}{\partial t} - \sum_{p=1}^n \frac{\partial u^j}{\partial x^p}\frac{\partial u^k}{\partial x^p}\right\} = 0, \tag{1.4}$$

$1 \le i,j,k \le m$, Γ^i_{jk} are the Christoffel symbols corresponding to the metric in (1.3) and summation over repeated indices is understood.

There has been a lot of research done regarding different aspects of harmonic maps [**1, 2, 3** and references therein]. Here we look at the behavior of some special solutions of (1.4) defined below.

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