

CAN MUTUALISM ALTER COMPETITIVE OUTCOME?: A MATHEMATICAL ANALYSIS

H.I. FREEDMAN AND BINDHYACHAL RAI

ABSTRACT. A model is described whereby two interacting competitors also interact with a mutualist. After examining equilibria and their stability, the question of “reversal of competitive outcome” due to the mutualism is investigated.

1. Introduction. Even in the general Kolmogorov model of two competitors, it is known (Albrecht et al. [1]) that the dynamics are trivial, i.e., all solutions approach an equilibrium. The same is true when two mutualists interact (Freedman et al. [5]).

The main purpose of this paper is to consider the situation when two interacting competitor populations each interact with a mutualist as well. We are particularly interested in determining criteria for persistence of all populations in the case when one of the competitors becomes extinct in the absence of its mutualist.

Kirlinger [9, 10] has shown for Lotka-Volterra models that the bistable case in two-species competition can be made permanent (uniformly persistent) by two predators, but never by one predator. In this paper we obtain similar results for more general competitive systems regulated by two mutualists.

Original models of mutualism were two dimensional (Dean [3], Freedman [4], Freedman et al. [5]). Three dimensional models where the mutualism arose due to the presence of either a predator or a competitor were first analyzed in Rai et al. [12] and expanded upon by Kumar and Freedman [11]. Since then other works involving competitor-competitor-mutualist interactions have been published (Freedman and Rai [6, 7]).

Received by the editors on July 13, 1992, and in revised form on January 20, 1993.

Research of the first author partially supported by the Natural Sciences and Engineering Research Council of Canada, Grant No. NSERC A4823.

Research of the second author partially supported by DAAD fellowship while visiting the Universität Kaiserslautern, Kaiserslautern, Germany, during the period from September 1 to November 30, 1992.