

**SOME GLOBAL QUALITATIVE ANALYSES  
OF A SINGLE SPECIES NEUTRAL DELAY  
DIFFERENTIAL POPULATION MODEL**

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**ABSTRACT.** In this paper a class of nonlinear nonautonomous single species neutral delay differential population models are introduced and analyzed. Sufficient conditions for positivity and boundedness of solutions, local and global stability of positive steady state, are established.

**1. Introduction.** The most frequently adopted single species population model takes the form

$$(1.1) \quad \dot{x}(t) = rx(t)[1 - x(t)/K], \quad x(0) > 0,$$

where  $r$  is the so-called intrinsic growth rate of the species  $x$ , and  $K$  is often referred to as the environment carrying capacity for  $x$ .  $r[1 - x(t)/K]$  is called the per capita growth rate of  $x$  at time  $t$ , which asserts that the growth rate is inhibited due to the self crowdedness effect.

If we take into account the fact that species  $x$  may have a gestation period of length  $\tau$ , then a more suitable expression of the per capita growth rate should be

$$(1.2) \quad r[1 - ax(t) + bx(t - \tau)]$$

with  $a - b = 1/K$ . The term  $rbx(t - \tau)$  reflects the birth rate due to the part of the population of age  $\tau$ . Equation (1.1) thus becomes

$$(1.3) \quad \dot{x}(t) = rx(t)[1 - ax(t) + bx(t - \tau)].$$

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