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EIGENVALUE ASYMPTOTICS FOR A NON-SELFADJOINT ELLIPTIC PROBLEM INVOLVING AN INDEFINITE WEIGHT

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ABSTRACT. Asymptotic formulae are established for the distribution functions of the real parts of the eigenvalues of a non-selfadjoint linear elliptic boundary value problem involving an indefinite weight function under more general conditions than hitherto considered.

1. Introduction. An inspection of the literature concerning the eigenvalue asymptotics for linear elliptic boundary value problems involving an indefinite weight function shows that all of the relevant work to date has been devoted to either selfadjoint problems or non-selfadjoint problems arising from perturbations of selfadjoint ones. We refer to [**4**–**7, 8, 10, 11**], and [**13, 14**] for further information. We are now going to focus our attention upon the eigenvalue asymptotics for a non-selfadjoint problem which does not arise from a perturbation of a self-adjoint one.

Accordingly, we shall be concerned here with the boundary value problem

$$Lu = \lambda\omega(x)u \quad \text{in } \Omega, \tag{1.1}$$

$$B_j u = 0 \quad \text{on } \Gamma \qquad \text{for } j = 1, \dots, m, \tag{1.2}$$

where L is a linear elliptic operator of order $2m$ defined in a bounded region $\Omega \subset \mathbf{R}^n, n \geq 2$, with boundary Γ, the B_j are linear differential operators defined on Γ, and ω is a real-valued function in $L^\infty(\Omega)$ which assumes both positive and negative values. Our assumptions concerning the problem (1.1–2) will be made precise in Section 2; and in particular we might mention that it will always be supposed here that $2m > n$ and that $1/\omega \in L^\infty(\Omega)$. As a consequence of our assumptions

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