ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 1, Winter 1995

ON THE DISCRETE RICCATI EQUATION AND ITS APPLICATIONS TO DISCRETE HAMILTONIAN SYSTEMS

LYNN H. ERBE AND PENGXIANG YAN

ABSTRACT. We obtain some comparison theorems for the discrete Riccati equation

$$\Delta W(t)+A(t)+B^*(t)W(t)+W(t)B(t)-B^*(t)W(t)B(t)$$
$$+(I-B(t)))^*W(t)(C^{-1}(t)+W(t))^{-1}W(t)(I-B(t))=0$$

and some applications to the discrete Hamiltonian difference system

$$\Delta y(t) = B(t)y(t+1) + C(t)z(t)$$
$$\Delta z(t) = -A(t)y(t+1) - B^*(t)z(t)$$

where A, B, and C are $d \times d$ matrix functions, $y(t)$, $z(t)$ are $d \times 1$ vectors and t takes on integer values in $[M-1, N+1]$.

1. Introduction. In [**5–8**] the present authors introduced the Hamiltonian vector difference system:

$$\begin{aligned} \Delta y(t) &= B(t)y(t+1) + C(t)z(t) \\ \Delta z(t) &= -A(t)y(t+1) - B^*(t)z(t) \end{aligned} \tag{1.1}$$

the corresponding matrix system

$$\begin{aligned} \Delta Y(t) &= B(t)Y(t+1) + C(t)Z(t) \\ \Delta Z(t) &= -A(t)Y(t+1) - B^*(t)Z(t) \end{aligned} \tag{1.2}$$

where $A(t), B(t), C(t), W(t), Y(t)$, and $Z(t)$ are $d \times d$ matrices with $A(t)$ and $C(t)$ Hermitian. We assume further that $C(t) > 0$ (positive definite) and $I - B(t)$ is invertible. Also, Δ denotes the forward

Received by the editors on September 23, 1992, and in revised form on January 6, 1993.
Key words. Discrete Riccati equation, comparison theorem, autonomous system, disconjugacy, Hamiltonian difference system.
Research supported by NSERC-Canada