ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 1, Winter 1995

SYMMETRIC PERIODIC SOLUTIONS OF RATIONAL RECURSIVE SEQUENCES

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ABSTRACT. We consider the rational recursive sequence

$$(*)\qquad x_{n+1} = \frac{a + \sum_{i=0}^{k-1} b_i x_{n-i}}{x_{n-k}}, \qquad n = 0, \pm 1, \pm 2, \dots$$

where

$$a \in (0, \infty) \quad \text{and} \quad b_0, \dots, b_{k-1} \in [0, \infty)$$

and show that, under appropriate hypotheses, when the linearized equation

$$Ey_{n+1} + Ey_{n-k} = \sum_{i=0}^{k-1} b_i y_{n-i}, \qquad n = 0, \pm 1, \pm 2 \dots$$

about the positive equilibrium E of $(*)$ has a periodic solution with minimal period $2(k+1)$, then $(*)$ also has a periodic solution with the same minimal period.

1. Introduction. Consider the rational recursive sequence

$$(1)\qquad x_{n+1} = \frac{a + \sum_{i=0}^{k-1} b_i x_{n-i}}{x_{n-k}}, \qquad n = 0, \pm 1, \pm 2, \dots ,$$

where

$$(2)\qquad a \in (0, \infty) \quad \text{and} \quad b_0, \dots, b_{k-1} \in [0, \infty).$$

Our aim in this paper is to show that, under appropriate hypotheses, when the linearized equation

$$(3)\qquad Ey_{n+1} + Ey_{n-k} = \sum_{i=0}^{k-1} b_i y_{n-i}, \qquad n = 0, \pm 1, \pm 2, \dots$$

Received by the editors on November 13, 1992.
Key words. Rational recursive sequence, periodic solution, symmetric solution, symmetric periodic solution.
1990 AMS *Subject Classification.* 39A12.