ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 1, Winter 1995

ULTIMATE BOUNDS AND GLOBAL ASYMPTOTIC STABILITY FOR DIFFERENTIAL DELAY EQUATIONS

YULIN CAO AND THOMAS C. GARD

ABSTRACT. We use an interval mapping method to produce a sequence of improved ultimate bounds for positive solutions of differential delay equation models for population growth. We obtain a general result for global asymptotic stability of a positive equilibrium as a consequence.

1. Introduction. We consider the population dynamics model with (possibly varying) time delay

$$\dot{x}(t) = x(t)f(x(t-\tau(t)),t). \tag{1.1}$$

Since $x(t)$ in (1.1) represents a population density, we restrict our attention to positive solutions of (1.1). (See Lemma 1 below.) Although (1.1) does not contain all biologically relevant differential delay equation models of population growth (Cushing [**2**], Freedman and Gopalsamy [**3**], Gurney, Blythe and Nisbet [**5**]), it is sufficiently general to include, for example, the modified logistic delay equation (1.1) with

$$f(x(t-\tau(t)),t) = a + bx(t-\tau) - cx^2(t-\tau), \tag{1.2}$$

treated recently by Gopalsamy and Ladas [**4**]. In this paper our main goal is to provide new checkable conditions for global asymptotic stability of the positive equilibrium of (1.1). To achieve this goal we first extend slightly one of our recent results [**1**, Proposition 1] which gives permanence for (1.1); this extension is Theorem 2 below. We then establish a refinement (Theorems 3 and 6) of Theorem 2 in which sharper estimates for the attracting set for positive solutions are obtained. This is accomplished by producing a sequence of improved estimates for the ultimate bounds for such solutions using an interval

Received by the editors on September 28, 1992, and in revised form on February 1, 1993.
Key words and phrases. Differential delay equations, global asymptotic stability, population dynamics.