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SPATIALLY DISCRETE NONLINEAR DIFFUSION EQUATIONS

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ABSTRACT. We consider spatially discrete nonlinear diffusion equations that are similar in form to the Cahn-Hilliard and Cahn-Allen equations. Since these equations are spatially discrete, solutions exist even for negative gradient energy coefficients. In order to study these equations analytically on finite subsets of one, two and three dimensional lattices, we propose a discrete variational calculus. It is shown that, under very general boundary conditions, these equations possess a gradient structure. We prove the existence of a global attractor and show that when all equilibria are hyperbolic the global attractor consists of the equilibria and the connecting orbits between the equilibria. The equilibria of specific one-, two- and three-dimensional equations are studied. We exhibit constant, two-periodic and three-periodic equilibrium solutions and study their stability properties. Numerical methods for solving the time dependent equations are proposed. We employ a fully implicit time integration scheme and solve the equations on a massively parallel SIMD machine. To take advantage of the structure of our problem and the data parallel computing equipment, we solve the linear systems using the iterative methods CGS and CGNR. Finally, we exhibit the results of our numerical simulations. The numerical results show the robust pattern formation that exists for different values of the parameters.

1. Introduction. In this paper we consider spatially discrete nonlinear diffusion equations that occur as models for binary alloys. These equations are truly spatially discrete and are not space discretized partial differential equations. In fact, for some of the parameter values that we consider there may not exist a well-posed PDE even in a weak sense. The differential equations we consider are analogous in form to the Cahn-Hilliard equation (see [**4**]) and the Cahn-Allen equation (see

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