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MODELS FOR DISEASES WITH EXPOSED PERIODS

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ABSTRACT. A general model for a disease without immunity against reinfection having arbitrary distributions of exposed and infective periods was formulated by Hethcote, Stech and van den Driessche [**5**]. They showed that for contact numbers exceeding 1, the endemic equilibrium is asymptotically stable if either the exposed period of the infective period is exponentially distributed or if both exposed and infective period have fixed length, and they conjectured that the endemic equilibrium is always asymptotically stable.

We show that the endemic equilibrium is asymptotically stable if the mean exposed period is less than the mean infective period, or if the contact number is sufficiently large, or if the exposed period distribution function is convex. However, we also show that for a more general type of model in which the infective period distribution can depend on the length of the exposed period it is possible to have instability of the endemic equilibrium and a Hopf bifurcation.

1. There are three main categories of simple models for the spread of communicable diseases, namely, S-I-R models with removal through recovery and immunity against reinfection, S-I-R models with removal through death caused by the disease, and S-I-S models with recovery but with no immunity against reinfection. The formulation of these models goes back to the three fundamental papers of Kermack and McKendrick [**8**]. Descriptions which may be easier to follow may be found in [**1, 2, 7**]. Variations such as an exposed period between infection by the disease and becoming infective, or a period of temporary immunity following recovery from the disease are most readily incorporated into the framework of these three basic categories of models by allowing the infectivity of an individual to depend on the time since infection [**3, 10**].

For each of the basic models there is a basic reproductive number or contact number R_0 depending on the rate of transmission of infection,

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