ROCKY MOUNTAIN
JOURNAL OF MATHEMATICS
Volume 25, Number 1, Winter 1995

OSCILLATORY PROPERTIES OF SOLUTIONS AND NONLINEAR DIFFERENTIAL EQUATIONS WITH PERIODIC BOUNDARY CONDITIONS

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To the memory of Geoffrey J. Butler
and to the dedication of his colleagues in Edmonton

1. Introduction. On September 3, 1979, returning from a meeting in Szeged, Geoffrey J. Butler visited Louvain-la-Neuve and delivered a lecture on *The Poincaré-Birkhoff theorem and periodic solutions of second order nonlinear differential equations.* In his talk, Butler described some of his recent work on the use of the Poincaré-Birkhoff fixed point theorem for the obtention of nontrivial T-periodic solutions of 'unforced' second order differential equations of the form

$$x'' + f(t,x) = 0,$$

when $f(t,0) = 0$ for all t and f is superlinear with respect to x, i.e.,

$$\frac{f(t,x)}{x} \longrightarrow +\infty \quad \text{as } |x| \longrightarrow \infty.$$

He conjectured in his lecture that this approach could be modified to be applicable to the 'forced' case, i.e., when $f(t,0) \not\equiv 0$, but never published any further paper about this conjecture (which is explicitly mentioned in [**6**]). His results on the unforced case [**4, 5**] make an extensive use of the *oscillatory* properties of the solutions of the differential equation, a topic to which Butler has masterly contributed in several directions (see, e.g., the analysis in [**13**]).

It may therefore be appropriate to show in these Butler lectures how the use of oscillatory properties of the solutions of ordinary differential equations may help in proving the existence of periodic solutions for ordinary differential equations. The recent results described here will, hopefully, convince the reader how fruitful and vivid are the ideas that

Received by the editors on September 22, 1992, and in revised form on December 22, 1992.