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AREA INTEGRAL ASSOCIATED WITH SINGULAR MEASURES ON THE UNIT SPHERE ON C^n

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1. Introduction. The purpose of this paper is to study some problems relating to the Lusin area integral [**8**]. In [**1**], P. Ahern and A. Nagel introduced a modified area integral, which is given by, for $0 < p < 2$,

$$G_p(f)^2(\xi) = \int_{\mathcal{A}_\alpha(\xi)} [|\nabla f(z)|^2 \rho(z)^{1-n+2(n-m)/p} + |\nabla_T f(z)|^2 \rho(z)^{-n+2(n-m)/p}]\, d\nu(z)$$

and they proved that if μ is a positive measure on the boundary of the unit ball, such that $\mu(B(\xi,\delta)) \le C\delta^m$, (hence μ may be singular) then the following singular area integral inequality, for every f in H^p, $1 < p < 2$,

$$||G_p(f)||_{L^p(d\mu)} \le C_p ||f||_{H^p}.$$

The proof proceeds in two steps. First they showed in [**1**] that the term involving the tangential part of the gradient is essentially dominated by the other term. To treat the other part they applied an analogue, for domains in C^n, of the tent space T^1_∞, which is introduced by R.R. Coifman, Y. Meyer and E. Stein [**2, 3**].

In this paper the result of Ahern-Nagel will be extended to the case $0 < p \le 2$. Here the main tool is not T^1_∞ space but T^p_2 space.

2. Preliminaries and terminologies. For two complex n vectors $z = (z_1, \dots, z_n)$ and $w = (w_1, \dots, w_n)$, the inner product $\langle z, w\rangle$ is given by $\langle z, w\rangle = \sum_{i=1}^n z_i \bar{w}_i$, and the corresponding norm will be $|z| = (\sum_{i=1}^n |z_i|^2)^{1/2}$. For ξ, η in the unit sphere S of the unit ball $B = \{|z| < 1\}$ and $\delta < 0$, let $\rho(\xi,\eta) = |1 - \langle \xi, \eta\rangle|$ and $B(\xi,\delta) = \{\eta \in S : \rho(\eta,\delta) = |1 - \langle \eta, \xi\rangle| < \delta\}$.

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