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ON AN ALTMAN TYPE FIXED POINT THEOREM ON CONVEX CONES

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1. Introduction. The principal result of this paper is a fixed point theorem on a convex cone in a Hilbert space. This result is obtained using the complementarity theory.

This fact is not surprising, since in our paper [**20**] we showed that there exist interesting implications from the fixed point theory to the complementarity theory, and reciprocally the complementarity theory can be used to obtain new fixed point theorems.

These relations are very interesting since the complementarity theory is in development and it has many and important applications in optimization, game theory, engineering, mechanics, elasticity theory, economics, etc., [**19, 13, 20**].

Let $(H, \langle , \rangle)$ be a separable Hilbert space. If $r > 0$ we denote $B_r = \{x \in H \mid ||x|| \leq r\}$ and $S_r = \{x \in H \mid ||x|| = r\}$.

In 1957 Altman proved the following fixed point theorem.

Theorem (Altman, [**1**]). *Let f be a weakly closed operator defined on B_r with range in H. If f maps the set B_r into a bounded set and the following condition is satisfied*

$$(A) \qquad \langle f(x), x \rangle \leq \langle x, x \rangle \quad \textit{for every } x \in S_r,$$

then f possesses a fixed point in B_r.

Another version of this theorem was proved by Shinbrot in 1965 [**32**] without the assumption that $f(B_r)$ is bounded but supposing that f is continuous from the weak to the weak topology.

Shinbrot's theorem has interesting applications to partial differential equations.

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