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THE POINT SPECTRA AND REGULARITY FIELDS OF NON-SELF-ADJOINT QUASI-DIFFERENTIAL OPERATORS

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ABSTRACT. In this paper the general ordinary quasi-differential expressions of nth order with complex coefficients are considered, and a number of results concerning the location of the point spectra and regularity fields of the operators generated by such expressions are obtained. Some of these are extensions or generalizations of those in the symmetric case in [**9, 10**] and [**11**], while others are new.

1. Introduction. The minimal operator T_0 and T_0^+ generated by a general quasi-differential expressions M and its formal adjoint M^+, respectively, form an adjoint pair of closed, densely defined operators in the underlying L_w^2-space, that is, $T_0 \subset (T_0^+)^*$. The operators which fulfill the role that the self-adjoint and maximal symmetric operators play in the case of a formally symmetric expression M are those which are regularly solvable with respect to T_0 and T_0^+. Such an operator S satisfies $T_0 \subset S \subset (T_0^+)^*$, and for some $\lambda \in \mathbf{C}$, $(S - \lambda I)$ is a Fredholm property of zero index; this means that S has the desirable Fredholm property that the equation $(S - \lambda I)u = f$ has a solution if and only if f is orthogonal to the solutions of $(S^* - \bar{\lambda} I)v = 0$, and furthermore the solution spaces of $(S - \lambda I)u = 0$ and $(S^* - \bar{\lambda} I)v = 0$ have the same finite dimension. This notion was originally due to Visik [**12**].

The main objectives of this paper are to investigate the location of the point spectra and regularity fields of general ordinary quasi-differential operators. Also, the results concerning the differential operator generalize all of those given in [**10, 11**] for the symmetric case and in [**8**] for the nonsymmetric case, by removing the condition on the regularity field.

We deal throughout with a quasi-differential expression M of arbitrary order n defined by a general Shin-Zettl matrix given in [**7**] and [**8**], and the minimal operator T_0 is generated by $(1/w)M[.]$ in $L_w^2(I)$,

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