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REPRESENTATIONS OF FINITE POSETS AND NEAR-ISOMORPHISM OF FINITE RANK BUTLER GROUPS

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Introduction. This paper contains a functorial interpretation of finite rank Butler groups, up to near isomorphism, as representations of finite *posets* (partially ordered sets) over $Z/p^m Z$. The representation setting clarifies the complexity and, in special cases, the structure of these groups. In particular, for $m = 1$ the theory of representations over a field is available. As an application, indecomposable almost completely decomposable *acd* groups with arbitrarily large finite rank and fixed typeset are constructed. Examples of this sort, so far as we know, are new. In the other direction, rigid uniform *acd* groups are classified up to near isomorphism by invariants in [**17**]. These invariants classify associated representations up to isomorphism.

A *Butler group* is a pure subgroup of a **cd** group, a finite direct sum of torsion-free abelian groups of rank 1 [**16**]. Each Butler group G has a finite typeset generating a finite distributive lattice T of *types* (isomorphism classes of rank-1 groups). Thus, G is in $\mathbf{B_T}$, the quasi-homomorphism category of Butler groups with types in T. There is a category equivalence from B_T to $\text{Rep}\,(Q, JI(T)^{op})$, the category of Q-representations of the opposite of the poset $JI(T)$ of join-irreducible elements of T [**14, 15**].

Group-theoretic properties are lost by passing to the quasi-homomorphism category. For example, an indecomposable group need not be *strongly indecomposable*, indecomposable relative to quasi-isomorphism. On the other hand, determining isomorphism of **acd** groups, Butler groups quasi-isomorphic to *cd* groups, leads to number-theoretic problems [**17**] that we wish to avoid.

Near-isomorphism, a generalization of genus class for lattices over Z-orders [**20**] is an equivalence relation on torsion-free abelian groups of finite rank lying between isomorphism and quasi-isomorphism.

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